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ELECTROHYDRODYNAMICS OF TWO-PHASE MEDIA WITH PARTICLES OF THE DISPERSED PHASE CHARGED BY AN ELECTRIC FIELD*

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A polydisperse two-phase system consisting of solid or liquid particles of the dispersed phase and gas with bipolar charge in an electric field is considered. Volume concentration of the dispersed phase is assumed low. Charging of particles by the capture of ions from the gas under the action of electric field is investigated. A model of such medium is constructed using concepts of continuous medium mechanics /1/ with allowance for variation of the charge of moving particles. Various methods are proposed for simplifying the equations of particles charging and motion. It is shown that, unlike in electrohydrodynamics of multiphase media without charging particles /2/, the mobility of particles moving under the effect of an electric field can change. Relaxation of particle charge and velocity behind a shock wave is investigated.

1. Charging of dispersed particles in a gas with bipolar charge. In two-phase media consisting of dispersed solid or liquid particles and charge gas, the particles may become charged by collecting ions from the surrounding gas. We begin the investigation of this phenomenon with the case of low concentration of dispersed particles by considering the charging of a spherical particle of radius a in gas with positive and negative ions whose concentration is also assumed low. The undistrubed (by the particle) velocity $\mathbf{v}$ of gas, the ion electric charge density $q_\pm$, and the electric field intensity $\mathbf{E}$ are assumed to be fairly slow varying functions of coordinates and time. We shall consider the relative motion of gas in a system of coordinates attached to the particle on the assumption of validity of the neglect in the Navier–Stokes equations of the following terms and quantities: the inertial terms and the Coulomb electric force, as small in comparison with the viscosity term; the ion diffusion at distances of order a as small in comparison with convective transport and drift due to the electric field effect; variation of the electric field intensity due to perturbation in the particle neighborhood, as small in comparison with its unperturbed value, and the conductivity and viscosity of gas, as small in comparison with respective particle parameters. The particle velocity relative to gas $\mathbf{u}$ and the unperturbed electric field intensity $\mathbf{E}$ are assumed in the considered case to be parallel to the same straight line.

Formulas for electric fluxes of positive and negative ions reaching the particle can be represented for various velocities and charges in the form /3,4/

$$J_\pm = (Q^\circ/\tau_\pm) J_\pm^*(u^*, Q^*), \quad Q^\circ = 3|\mathbf{E}|a^2, \quad \tau_\pm = (4\pi b_\pm q_\pm)^{-1} \tag{1.1}$$
$$u^* = u/(b_p^\circ|\mathbf{E}|), \quad Q^* = Q/Q^\circ, \quad b_p^\circ = Q^\circ/(6\pi\mu a)$$

$$\begin{aligned} &J_\pm^* = 0, \quad u^* \lesseqgtr b_\pm/b_p^\circ, \quad Q^* \gtreqless \pm 1 \\ &\qquad u^* \gtreqless b_\pm/b_p^\circ, \quad Q^* \gtreqless 0 \\ &J_\pm^* = -Q^*, \quad u^* \lesseqgtr b_\pm/b_p^\circ, \quad Q^* \lesseqgtr \mp 1 \\ &\qquad u^* \gtreqless b_\pm/b_p^\circ, \quad Q^* \lesseqgtr 0 \\ &J_\pm^* = \pm{}^1/_4(1 \mp Q^*)^2, \quad u^* \lesseqgtr b_\pm/b_p^\circ, \quad |Q^*| < 1 \end{aligned} \tag{1.2}$$

where Q and Q° are the particle charge and its characteristic value, respectively, τ_+ (τ_-) are characteristic times of particle charge variation caused by the capture of positive (negative) ions, b_p° is the characteristic value of particle mobility, u is the projection of particle velocity on the direction of $\mathbf{E}$, u^*, Q^* are dimensionless values of u, Q, $b_+ > 0$ and $b_- < 0$ are the mobilities of positive and negative ions, respectively. The gas permittivity is assumed equal unity, and the super- and subscripts denote J_+ and J_-, respectively. Formulas (1.1) and (1.2) are obtained by analyzing ion streamlines at low Reynolds numbers $\mathrm{Re}_p = 2a\rho|u|/\mu \ll 1$ (ρ and μ are the density and dynamic viscosity of gas) in a coordinate system attached to the particle.

The equations that define the charging and motion of the particle at $\mathrm{Re}_p \ll 1$ in dimensionless variables Q^*, u^* are of the form

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$$dQ^*/dt = J_+^*/\tau_+ + J_-^*/\tau_-, \quad du^*/dt = (Q^* - u^*)/\tau_v \tag{1.3}$$

where m is the mass of the particle, t is the time, and $\tau_v = m/(6\pi\mu a)$ is the characteristic time of particle velocity change. The unperturbed values of $\mathbf{v}$, $q_\pm$, $\mathbf{E}$ are assumed to be fairly slow varying functions of coordinates and time.

We define the particle mobility b_p by the equality $b_p = Q/(6\pi\mu a)$. For particles of radius $a \lesssim 10^{-5}$ m we have the inequality $|b_p| \ll |b_\pm|$ which is satisfied, even when the quantity $|Q|/a^2$ reaches the breakdown value of voltage of the electric field in air. Hence, when $u^* \gtrsim b_+/b_p^\circ > 0$ and $u^* \lesssim b_-/b_p^\circ < 0$, the term Q^* is the second of Eqs.(1.3) can be neglected, as small in comparison with u^*

$$|Q^*/u^*| \lesssim |Qb_p^\circ/(Q^\circ b_\pm)| = |b_p/b_\pm| \ll 1$$

As the result, system (1.3) splits in the indicated region of u^* values.

The qualitative behavior of phase trajectories of system (1.3) is shown in Fig.1. Motion along them, indicated by arrows, corresponds to increasing time. The derivatives dQ^*/du^* and dQ^*/dt vanish along the following three straight-line segments:

$$Q^* = Q_S^* \equiv \frac{\sqrt{\tau_-} - \sqrt{\tau_+}}{\sqrt{\tau_-} + \sqrt{\tau_+}}, \quad \frac{b_-}{b_p^\circ} < u^* < \frac{b_+}{b_p^\circ} \tag{1.4}$$

$$Q^* = Q_\pm^* \equiv \pm\left(1 + 2\frac{\tau_\pm}{\tau_\mp}\right) \mp \left[\left(1 + 2\frac{\tau_\pm}{\tau_\mp}\right)^2 - 1\right]^{1/2}, \quad u^* \lessgtr \frac{b_\mp}{b_p^\circ} \tag{1.5}$$

which simultaneously are sections of phase trajectories. Parameters Q_S^*, $Q_\pm^*$ obviously satisfy the inequalities $|Q_S^*| < 1$, $0 \lessgtr Q_\pm^* \lessgtr \pm 1$. On the straight line $Q^* = u^*$ (represented in Fig.1 by dashes) the derivative dQ^*/du^* becomes infinite along phase trajectories, while the derivative du^*/dt vanishes. System (1.3) has a singular point S at coordinates $u^* = Q^* = Q_S^*$ in the form of a steady node. That point is approached by all trajectories along paths for which $dQ^*/du^* = [1 - \tau_h/\sqrt{\tau_+\tau_-}] \equiv k$ or $dQ^*/du^* = 0$. If $\tau_h > \sqrt{\tau_+\tau_-}$, only two trajectories approach point S along the first of these paths, while along the other it is approached by the infinite number of remaining trajectories. However, when $\tau_h < \sqrt{\tau_+\tau_-}$, the contrary is true. This case is represented in Fig.1, where the thin straight line corresponds to $(Q^* - Q_S^*) = k(u^* - u_S^*)$ and is tangent to an infinite number of phase trajectories. The singular point S evidently represents a stable steady state of the particle for which $Q_S = Q^\circ Q_S^*$, $u_s = b_p^\circ |\mathbf{E}| \cdot Q_S^* = b_p |\mathbf{E}|$. If at the initial instant of time Q, u differed from their steady state values, they relax in the course of time to those values. The relaxation time of particle charge is of order $\max(\tau_+, \tau_-)$ and of its velocity of order τ_h.

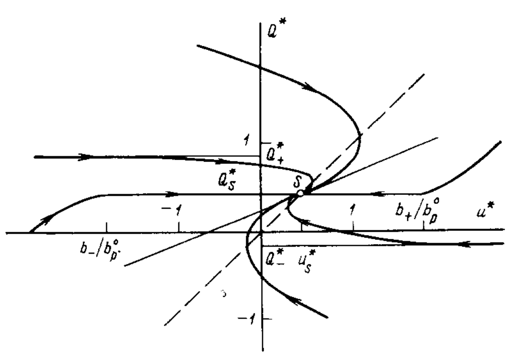

Fig.1

Note that in the steady state the sum of friction and electrical forces acting on the particle is zero. Hence the condition stated above that vectors of particle velocity relative to gas and of the electric field intensity must be parallel, is alway satisfied.

We divide the phase plane (Q^*, u^*) in three regions: $u^* < b_-/b_p^\circ$, $b_-/b_p^\circ < u^* < b_+/b_p^\circ$, $b_+/b_p^\circ < u^*$. Functions $J_\pm^*(Q^*, u^*)$ defined by equalities (1.2) are independent of u^* inside each of these regions, but change when passing from one region to another. This enables us to integrate system (1.3) and obtain, for the determination of its phase trajectories, the following relations:

$$u^* = f(Q^*)\left\{\int\left[f(Q^*)\left(\frac{\tau_h}{\tau_+}J_+^* + \frac{\tau_h}{\tau_-}J_-^*\right)\right]^{-1} Q^* dQ^* + C\right\} \tag{1.6}$$

$$f(Q^*) = \exp\left[-\int\left(\frac{\tau_h}{\tau_+}J_+^* + \frac{\tau_h}{\tau_-}J_-^*\right)^{-1} dQ^*\right] \tag{1.7}$$

Inside each of these regions the quantity C is constant along phase trajectories (but, obviously, is not the same on different trajectories). At transition from one region to another C changes, and is determined by the condition of continuity of phase trajectories. Functions $J_\pm^*(Q^*)$ and $f(Q^*)$ also change at that stage. The indefinite integrals in equalities

(1.6) and (1.7) denote any fixed antiderivatives of integrands when $-\infty < Q^* < +\infty$. Function $f(Q^*)$ is defined in terms of elementary functions of different form in each of the indicated regions.

2. Equations of electrohydrodynamics of two-phase media. Consider a two-phase polydisperse medium consisting of a gas with bipolar charge and dispersed spherical particles in an electric field. We assume the dispersed phase volume concentration to be small, and viscosity and the thermal conductivity of gas as immaterial in the investigation of its averaged motion. The averaged motions of gas, ions and particles can be defined on these assumptions in terms of continuous medium mechanics by equations of the form

$$\partial\rho/\partial t + \operatorname{div}\rho\mathbf{v} = 0, \quad \partial n_p/\partial t + \operatorname{div} n_p\mathbf{v}_p = 0 \tag{2.1}$$

$$\partial Q/\partial t + (\mathbf{v}_p\nabla)Q = J_+ + J_-, \quad \partial q_\pm/\partial t + \operatorname{div}\mathbf{j}_\pm = -\int J_\pm n_p da \tag{2.2}$$

$$\partial\rho\mathbf{v}/\partial t + \operatorname{div}\rho\mathbf{v}\mathbf{v} = -\nabla p + (q_+ + q_-)\mathbf{E} - \int n_p\mathbf{f}da \tag{2.3}$$

$$m(\partial\mathbf{v}_p/\partial t + (\mathbf{v}_p\nabla)\mathbf{v}_p) = \mathbf{f} + Q\mathbf{E}, \quad \mathbf{j}_\pm = q_\pm\mathbf{v} + q_\pm b_\pm\mathbf{E} \tag{2.4}$$

$$\partial\rho c_v T/\partial t + \operatorname{div}\rho c_v T\mathbf{v} = -p\operatorname{div}\mathbf{v} + (b_+q_+ + b_-q_-)E^2 - \int n_p[\mathbf{f}(\mathbf{v}_p - \mathbf{v}) + W]da, \quad p = \rho RT \tag{2.5}$$

$$m_p[(\partial/\partial t)c_pT_p + (\mathbf{v}_p\nabla)c_pT_p] = W \tag{2.6}$$

$$\operatorname{div}\mathbf{E} = 4\pi\left(q_+ + q_- + \int n_pQda\right), \quad \operatorname{rot}\mathbf{E} = 0 \tag{2.7}$$

where ρ, $\mathbf{v}$, p, T are the density, velocity, pressure, and temperature of gas, R is the gas constant, c_v is the specific heat at constant volume of gas, $n_p(a)\,da$ is the concentration of dispersed particles of radius a from the interval $[a, a + da]$, $\mathbf{v}_p(a)$, $T_p(a)$, $Q(a)$, $m = 4\pi\rho_p{}^\circ a^3/3$ are the velocity, temperature, charge, and mass of dispersed particles of radius a, $\rho_p{}^\circ$, c_p are the density and specific heat of particle material, $q_\pm$, $j_\pm$ are densities of the electric charge and current of positive and negative ions, respectively, $J_\pm(a)$ are the electric currents of positive and negative ions reaching a particle of radius a, $\mathbf{f}(a)$ is the resistance offered by gas to the motion of particle of radius a, $W(a)$ is the heat flux from gas to the particle of radius a, and $\mathbf{E}$ is the electric field intensity. The above parameters represent values averaged over physically infinitely small volumes of a reasonably large number of dispersed particles.

The system of Eqs.(2.1)–(2.7) is presented in the form of electrohydrodynamic approximation /5/ without allowance for ionization and recombination processes.

The first of Eqs.(2.1) is the equation of continuity of the gas phase density and the second represents in differential form the law of conservation of the number of particles of each radius. The first of Eqs.(2.2) is the equation of charge variation of dispersed particles. In the case in which at every point of space vectors $\mathbf{u} \equiv \mathbf{v}_p - \mathbf{v}$ and $\mathbf{E}$ are collinear, i.e. parallel or directed toward each other, formulas (1.1) and (1.2) can be used for expressing $J_\pm$ appearing in the right-hand side of the first of Eqs.(2.2). Parameters $\mathbf{u} \equiv \mathbf{v}_p - \mathbf{v}$, $\mathbf{E}$, $q_\pm$ are to be understood as the respective mean characteristics of the considered polydisperse medium defined at the given point. The collinearity condition for vectors $\mathbf{u}$ and $\mathbf{E}$ is always satisfied in the practically important case in which it is possible to neglect the effect the inertia of dispersed particles on their motion. The above condition is also valid in one-dimensional flows. The second of Eqs.(2.2) defines the variation of electric charge density of positive and negative ions. Equation (2.3) is the equation of motion of gas. The first of Eqs.(2.4) defines the motion of dispersed particles and the second (Ohm's law) is a simplified equation of motion of ions. Concentration of ions and dispersed particles is assumed low, so that the force of their interaction can be neglected, as small in comparison with that of interaction with gas. Equation (2.5) defines the influx of heat to the gas and ion mixture. The gas and ions are assumed to be at the same temperature. Equation (2.6) defines the temperature variation of dispersed particles induced by the heat exchange with gas. In the last two equations the energy exchange between ions and particles was assumed small and has been neglected. Equations (2.7) define the electric field.

The motion of a two-phase monodisperse medium, consisting of charged gas and dispersed particles of uniform size, in an electric field, can be defined by the system of Eqs.(2.1)–(2.7) in which the second of Eqs.(2.2) and in Eqs.(2.3), (2.5) and (2.7) integration with respect to particle radius are omitted. Parameter n_p is to be taken as the concentration of particles.

To close the system of equations it is necessary to specify the expressions for friction force $\mathbf{f}$ and the heat flux W. They can be represented in the form /6/

$$\mathbf{f} = {}^1\!/_2 \pi C_x(\mathrm{Re}_p)\, a^2 \rho\, |\mathbf{v} - \mathbf{v}_p|\, (\mathbf{v} - \mathbf{v}_p), \quad \mathrm{Re}_p \equiv 2a\rho\, |\mathbf{v} - \mathbf{v}_p|/\mu \tag{2.8}$$

$$W = 2\pi\, \mathrm{Nu}_p\, (\mathrm{Re}_p,\ \mathrm{Pr})\, a\varkappa\, (T - T_p), \quad \mathrm{Pr} = \mu c_v/\varkappa \tag{2.9}$$

where C_x, Nu_p, Re_p are, respectively, the drag coefficient, the Nusselt number, and the Reynolds number of the dispersed particle, Pr is the Prandtl number of gas, and μ and $\varkappa$ are, respectively, the coefficient of dynamic viscosity and thermal conductivity of gas. For functions $C_x(\mathrm{Re}_p)$, $\mathrm{Nu}_p(\mathrm{Re}_p, \mathrm{Pr})$ we have valid the following formulas /6,7/:

$$C_x = \frac{24}{\mathrm{Re}_p}, \quad \mathrm{Nu}_p = \begin{cases} 2, & \mathrm{Re}_p\mathrm{Pr} \ll 1 \\ 0.98 \cdot \sqrt[3]{\mathrm{Re}_p\mathrm{Pr}}, & \mathrm{Re}_p\mathrm{Pr} \gg 1 \end{cases} \quad \mathrm{Re}_p \ll 1 \tag{2.10}$$

$$\begin{gathered} C_x = 24\mathrm{Re}_p^{-1}\left(1 + {}^1\!/_6\, \mathrm{Re}_p^{2/3}\right), \quad 1 \lesssim \mathrm{Re}_p < 400 \\ \mathrm{Nu}_p = 2 + 0.6\, \mathrm{Re}_p^{1/2}\mathrm{Pr}^{1/3}, \quad 1 \lesssim \mathrm{Re}_p < 7 \cdot 10^4, \quad 0.6 < \mathrm{Pr} < 400 \end{gathered} \tag{2.11}$$

of which (2.10) is theoretical and (2.11) experimental.

3. Simplification of derived equations. The system of Eqs.(2.1)—(2.7) is fairly complex. Let us consider the possibility of replacing the complete equation of motion (the first of Eqs.(2.4)) of particles by Ohm's law for particles, with their temperature assumed equal to that of gas. We denote by $\tau_v = m/(6\pi\mu a)$ and τ the characteristic relaxation time of the dispersed particle velocity and the characteristic time of the problem, respectively, i.e. the minimal characteristic time of variation of defining parameters. The ratio $\tau_v/\tau = m/(6\pi\mu a\tau) = \mathrm{St}$ determines the effect of inertia on the motion of dispersed particles, and is called the Stokes number. If it is small, we can neglect the inertia terms in the equation of motion of dispersed particles. With allowance for equalities (2.8), (2.10), and (2.11) that equation assumes the form

$$\begin{gathered} \mathbf{v}_p = \mathbf{v} + b_p\mathbf{E} + B(\mathbf{v} - \mathbf{v}_p), \quad b_p = Q/(6\pi\mu a) \\ B = 0,\ \mathrm{Re}_p \ll 1; \quad B = {}^1\!/_6\, \mathrm{Re}_p^{2/3},\ 1 \lesssim \mathrm{Re}_p \lesssim 400 \end{gathered} \tag{3.1}$$

which is obtained by eliminating first force $\mathbf{f}$ from Eq.(2.3) of gas motion, using formula (2.4).

In conformity with equality (3.1), velocity $\mathbf{u} \equiv \mathbf{v}_p - \mathbf{v}$ of dispersed particles (of any radius a) relative to the gas and the electric field intensity $\mathbf{E}$ are everywhere directed along one straight line. Hence formulas (1.1) can be used for defining currents $J_\pm$. Note that the particle mobility b_p in the simplified equation (3.1) of particle motion, unlike in the conventional electrohydrodynamics of multiphase media /2/ (without allowance for the charging of dispersed phase particles), is not a constant coefficient. In conformity with the second of equalities (3.1) it depends on the particle charge whose variation is defined by the differential equation (2.2). Below, we call the approximation in which the equation of motion of particles is of the form (3.1) as the diffusion approximation.

It follows from Eqs.(2.6) and (2.9) that the characteristic relaxation time of particle temperature is $m_p c_p/(2\pi\, \mathrm{Nu}_p a\varkappa) \equiv \tau_T$. If $\tau_T/\tau \ll 1$, it is possible to use the simplified formula $T - T_p = 0$ instead of Eqs.(2.6) and (2.9) for determining the temperature of particles. It is then necessary to eliminate, as a preliminary, the heat flux W from Eq.(2.5) of heat flow to the gas, using equality (2.6).

Let us indicate the conditions under which Eq.(2.2) of particle charging can be replaced by an algebraic relation. We introduced in Sect.1 the characteristic times $\tau_\pm = (4\pi b_\pm q_\pm)^{-1}$ of particle charge variation due to capture of positive and negative ions. The ratio of the characteristic values of terms in the left-hand side of the first of Eqs.(2.2) to that of ion currents $J_\pm$ reaching a particle does not exceed $\tau_\pm/\tau$ in the order of magnitude. When this ratio is small, the terms in the left-hand side of the first of Eqs.(2.2) can be neglected, and the equation then reduces to the form

$$J_+ + J_- = 0 \tag{3.2}$$

We denote by $\tau_\pm'$ the characteristic times of variation of positive and negative ion charge density due to the deposition of ions on particles. Obviously

$$\tau_\pm' \sim \left| q_\pm \left[\int J_\pm n_p\, da\right]^{-1} \right| \sim \left(4\pi \left| b_\pm \int Q^\circ n_p\, da \right|\right)^{-1}$$

From the condition $\tau_\pm \ll \tau$ and definition of the problem characteristic time τ follows that $\tau_\pm \ll \tau \lesssim \tau_\pm'$. Substituting into it the expressions for $\tau_\pm$, $\tau_\pm'$, we obtain

$$|q_\pm| \gg \left| \int Q^\circ n_p\, da \right|$$

Hence in the case when the algebraic equation (3.2) is used instead of the differential equation (the first of Eqs.(2.2)) of particle charging, the charge density of positive and negative ions is considerably higher than the characteristic mean charge density of dispersed particles. Hence the equations of ion electric charge (the second of Eqs.(2.2)) remain valid also when the simplified equation (3.2) is used for defining particle charging. Substituting into it expressions (1.1) for currents $J_{\pm}$, we obtain for the dependence of dispersed particles charge on variables $\mathbf{u} \equiv \mathbf{v}_p - \mathbf{v}, \mathbf{E}, q_{\pm}$ the formula

$$Q = Q^\circ Q_S^* = 3|\mathbf{E}|a^2 \frac{\sqrt{b_+q_+} - \sqrt{b_-q_-}}{\sqrt{b_+q_+} + \sqrt{b_-q_-}}, \quad b_-E^2 < (\mathbf{u}\cdot\mathbf{E}) < b_+E^2 \tag{3.3}$$

$$Q = Q^\circ Q_\pm^* = 3|\mathbf{E}|a^2 \left\{\pm\left(1 + 2\frac{b_\mp q_\mp}{b_\pm q_\pm}\right) \mp \left[\left(1 + 2\frac{b_\mp q_\mp}{b_\pm q_\pm}\right)^2 - 1\right]^{1/2}\right\}, \ (\mathbf{uE}) \lessgtr b_\mp E^2$$

Since $\mathbf{u} \sim b_p\mathbf{E}$, $|b_p| \ll |b_\pm|$, $b_\pm \gtrless 0$, the first of equalities (3.3) holds in the case of diffusion approximation.

4. Shock waves. Consider steady electrohydrodynamic flows with shock waves of a two-phase medium, when the charge of dispersed particles can vary owing to ion deposition on them. Since the density of gas is considerably lower than that of particles material, the parameters n_p, $\mathbf{v}_p$, T_p of particles in a strong shock wave can be assumed continuous /8/. Jumps of the electric charge of particles and of densities of ion electric currents satisfy the relations $\{Q\} = \omega_+ + \omega_-$, $\{\mathbf{j}_\pm\}\cdot\mathbf{v} = -\int w_\pm n_p (\mathbf{v}_p\cdot\mathbf{v})\, da$, where $\{A\} = A_2 - A_1$, subscripts 1 and 2 denote values of parameter A ahead and behind the shock wave front, respectively, $\mathbf{v}$ is the vector of the normal to the shock wave, and $\omega_\pm(a)$ is the change of electric charge of particle of radius a at its intersection of the shock wave as the result of capture by the particle of positive (ω_+) or negative (ω_-) ions concentrated in the shock wave. If the surface charge $\sigma_+(\sigma_-)$ of positive (negative) ions in the shock wave is zero, then $\omega_+ = 0$ ($\omega_- = 0$). The remaining relations at the shock wave are the same as in electrohydrodynamics of single-phase media /5/, with the surface density of particles in the shock wave equal zero. The quantities $\omega_\pm$ must be specified on the basis of analysis of particle interaction with ions that constitute the surface charge $\sigma_\pm$.

Below, we consider the case in which the effect of particles on the motion of gas, ions, and electric field distribution can be neglected, the flow is one-dimensional, the shock wave is plane, vectors $\mathbf{v}, \mathbf{v}_p, \mathbf{E}$ are orthogonal to it, and $(\mathbf{v}\cdot\mathbf{v}_p) > 0$, $\{Q\} = 0$. The inequalities $\tau \equiv L/v_p \gg \tau_v, \tau \gg \tau_\pm$ where L is the minimal characteristic length of variation of parameters $\mathbf{v}, \mathbf{E}, q_\pm$, are assumed to be satisfied ahead and behind the shock wave front. This enables us to use the simplifed equations (3.1) and (3.3) for the determination of particles velocity $\mathbf{v}_p$ and charge Q ahead and at some distance behind the shock wave. The relaxtion zone of thickness $l \sim v_p \max(\tau_v, \tau_\pm) \ll L$ is immediately behind the shock wave. In that zone parameters $\mathbf{v}_p, Q$ change from $\mathbf{v}_{p1}, Q_1$ to $\mathbf{v}_{p2}, Q_2$ determined, respectively, by formulas (3.1) and (3.3) for $\mathbf{v} = \mathbf{v}_1, \mathbf{E} = \mathbf{E}_1, q_\pm = q_{\pm 1}$ and $\mathbf{v} = \mathbf{v}_2, \mathbf{E} = \mathbf{E}_2, q_\pm = q_{\pm 2}$ (since $l \ll L$, parameters $\mathbf{v}, \mathbf{E}, q_\pm$ inside that zone can be considered as constants at their values directly behind the shock wave). When $\mathrm{Re}_p \ll 1$), the parameter change $Q, \mathbf{v}_p$ is determined by the equations

$$v_{px}\, \partial Q/\partial x = J_+ + J_-, \quad v_{px}\partial u/\partial x = -6\pi\mu a u + QE_2, \quad u \equiv v_{px} - v_{x2} \tag{4.1}$$

The axis x of this coordinate system is directed along vector $\mathbf{E}_2$.

We pass in Eqs.(4.1) to dimensionless variables

$$Q^* = Q/Q^\circ, \quad u^* = u/(b_p^\circ \mathbf{E}_2) \quad (Q^\circ \equiv 3E_2a^2, \ b_p^\circ = Q^\circ/(6\pi\mu a))$$

In the plane Q^*, u^* the phase trajectories coincide with the trajectories of system (1.3). Hence all findings of Sect.1 (Fig.1) relative to the behavior of the latter are also valid for the system of Eqs.(4.1) that define the charge and velocity relaxation of particles behind the shock wave in a gas with bipolar charge.

5. Weak discontinuities. Consider the surfaces of a weak discontinuity in the electrohydrodynamics of two-phase monodisperse media, when the dispersing phase is an inviscid non-heat-conducting gas with bipolar charge. We direct the x axis along the normal to the discontinuity surface and denote by c the discontinuity propagation velocity relative to gas. Let the continuity of derivatives with respect to x and t in Eqs.(2.1)—(2.7) be disrupted at a weak discontinuity. Their jumps $\{\partial/\partial t\}, \{\partial/\partial x\}$ are obviously linked among themselves by the relation $\{\partial/\partial t\} = -(v_x + a)\{\partial/\partial x\}$. Using a reasoning similar to that in conventional electrohydrodynamics /5/, we obtain

$$c_1 = 0, \quad c_{2,3} = \pm[p(c_v + R)/(c_v\rho)]^{1/2}, \quad c_{4,5} = b_\pm E_x, \quad c_6 = u_x = v_{px} - v_x$$

for the propagation velocities of weak discontinuity surfaces, and the relations for jumps of derivatives on them.

Velocities $c_1, \ldots, c_5$ are the same as in electrohydrodynamics of homogeneous media /5/. On surfaces of weak discontinuities propagating at these velocities the derivatives of the dispersed phase parameters n, Q, T_p, $\mathbf{v}_p$ are continuous, while those of remaining parameters are interconnected by conventional electrohydrodynamic relations /5/. The surface of a weak discontinuity propagating at velocity c_6 obviously moves together with the dispersed particles. Along it we have arbitrary jumps $\{\partial n_p/\partial x\}$, $\{\partial Q/\partial x\}$, $\{\partial T_p/\partial x\}$, while the derivatives of remaining parameters, including particle velocity $\mathbf{v}_p$, are continuous.

If formula (3.1) is used as the equation of motion of particles, then with $\mathrm{Re}_p \ll 1$ we have $c_6 = u_x = b_p E_x$. In that case the jump of particle concentration derivative on the surfaces of weak discontinuities propagating at velocities $c_{2,3}$ is nonzero. $\{\partial n_p/\partial x\} = n_p (c - b_p E_x)^{-1} \{\partial v/\partial x\} \neq 0$, while on the weak discontinuity surface for which $c = c_6$ the derivatives of particle charge Q are continuous. The remaining formulas for jumps of derivatives remain the same.

6. Damping of small perturbations in the dispersed phase. Consider the propagation of small perturbations of dispersed phase parameters in a monodisperse medium consisting of charged gas and liquid or solid particles, when the effect of the latter on the motion of gas and ions, and also, on the electric field distribution can be neglected (the respective "interaction" parameters are small). We assume the gas velocity and temperature, density of ion electric charge, and the electric field intensity to be known functions of coordinates and time. The dispersed phase flow is then defined by the second of Eqs.(2.1), the first of Eqs. (2.2) and (2.4), and Eq.(2.6). Let us assume that the quantities $J_\pm$, $\mathbf{f}$, W, which define in these equations the interaction of phases, are determined by the formulas

$$\frac{J_\pm}{3|\mathbf{E}|a^2} = \pm\frac{1}{4\tau_\pm}\left(1 \mp \frac{Q}{3|\mathbf{E}|a^2}\right)^2, \quad \mathbf{f} = 6\pi\mu a(\mathbf{v} - \mathbf{v}_p), \quad W = 4\pi\varkappa a(T - T_p) \tag{6.1}$$

Let A be any of the dispersed phase parameters n_p, $\mathbf{v}_p$, Q, T_p which we represent in the form $A = A^\circ + A'$, where A° represents the unperturbed parameter and A' a small perturbation ($|A'| \ll |A^\circ|$). We shall consider short-wave perturbations for which the characteristic length λ of A' variation is considerably smaller than the characteristic length L of variation of the unperturbed value A°. Then, using the above equations for n_p, $\mathbf{v}_p$, Q, T_p, neglecting smalls of order A'^2, λ/L, and taking into account that these equations must be also satisfied for the unperturbed values n_p°, $\mathbf{v}_p^\circ$, Q°, T_p°, we obtain the relations

$$Dn_p' + n_p^\circ \nabla \mathbf{v}_p' = 0, \quad D\mathbf{v}_p' + \tau_v^{-1}\mathbf{v}_p' - (\mathbf{E}/m)Q' = 0 \tag{6.2}$$
$$DT_p' + \tau_T^{-1}T_p' = 0, \quad DQ' + \tau_Q^{-1}Q' = 0, \quad D \equiv \partial/\partial t + \mathbf{v}_p^\circ \cdot \nabla$$
$$\tau_v^{-1} = 6\pi\mu a/m, \quad \tau_T^{-1} = 4\pi\varkappa a/(mc_v), \quad \tau_Q^{-1} = \frac{1}{2}\left[\frac{N_+}{\tau_+} + \frac{N_-}{\tau_-}\right], \quad N_\pm \equiv \left(1 \mp \frac{Q^\circ}{3|\mathbf{E}|a^2}\right)$$

We seek a solution of this system of the form $A' = \mathrm{Re}\,[A^* \exp(i(\mathbf{kx} - \omega t))]$, where A^* is a slowly varying function of coordinates and time ($Lk \gg 1$, $\tau\omega \gg 1$, and L, τ are the characteristic length and the time of A^* variation). Substitution of this expression for functions A' into system (5.2), with the derivatives of A^* neglected, yields for the determination of amplitudes of A^* a system of homogeneous linear algebraic equations. Equating to zero its determinant we obtain the dispersion equation

$$\Omega(\Omega + i/\tau_v)^3(\Omega + i/\tau_T)(\Omega + i/\tau_Q) = 0, \quad \Omega \equiv \omega - (\mathbf{v}_p^\circ \cdot \mathbf{k})$$

whose solution is of the form $\Omega_1 = 0$, $\Omega_{2,3,4} = -i/\tau_v$, $\Omega_5 = -i/\tau_T$, $\Omega_6 = -i/\tau_Q$.

Thus in the considered here case there exist six types of small harmonic perturbations. Since $\partial\omega/\partial\mathbf{k} = \mathbf{v}_p^\circ$, all of them propagate at the unperturbed particle velocity $\mathbf{v}_p^\circ$. Perturbation of the first type, for which $n_p' \neq 0$, $\mathbf{v}_p' = T_p' = Q' = 0$, does not dampen or increase. Perturbations of the second, third, and fourth types, for which $n_p' \neq 0$, $\mathbf{v}_p' \neq 0$, $T_p' = Q' = 0$, decrease with decrement $1/\tau_v$. Perturbation of the fifth type, for which $T_p' \neq 0$, $n_p' = \mathbf{v}_p' = Q' = 0$, decrease with decrement $1/\tau_T$. Finally, perturbations of the sixth type, for which $n_p' \neq 0$, $v_p' \neq 0$, $Q' \neq 0$, $T_p' = 0$, decrease with decrement $1/\tau_Q$. If $Q^\circ = 3|\mathbf{E}|a^2 Q_S^*$, where Q_S^* is determined by the equality (1.4), then from the formula for τ_Q we have $\tau_Q = \sqrt{\tau_+\tau_-}$.

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